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#### Contents

- 1. Djemaia Bensikaddour and Lakehal Belarbi Minimal translation surfaces in Lorentz Heisenberg 3-space with flat metric, pp. 1-14. [PDF]
- 2. Braj Bushan Chaturvedi and Pankaj Pandey Existence of generalized mixed super quasi-constant curvature, pp. 15-25. [PDF]
- 3. Pratyay Debnath Almost pseudo product structure, pp. 26-37. [PDF]
- 4. Harry Gingold, Yotam Gingold and Salah Hamad A spherical projection of a complex Hilbert space is conformal iff it is the stereographic projection, Pp. 38-71. [PDF]
- 5. Brijesh Kumar Gupta, Braj Bhushan Chaturvedi and Mehraj Ahmad Lone, On Ricci semi-symmetric mixed super quasi-Einstein Hermitian manifold, pp. 72-82. [PDF]
- 6. Cristina Elena Hretcanu and Adara Monica Blaga Submanifolds in metallic Riemannian manifolds, pp. 83-97. [PDF]
- 7. Shyamal Kumar Hui, Laurian-Ioan Piscoran and Akshoy Patra On a gradient Ricci soliton in the plane, pp. 98-105. [PDF]
- 8. Sangeet Kumar Warped product semi-transversal lightlike submanifolds of indefinite nearly Kaehler manifolds, pp. 106-118. [PDF]
- 9. Yoshio Matsuyama Complete totally real submanifolds of a complex projective space, pp. 119-125. [PDF]
- 10. Fusun Ozen Zengin and Ayse Yavuz Tasci Some properties of pseudo conharmonically symmetric Riemannian manifolds, pp. 126-137. [PDF]
- 11. Ganesh Prasad Pokhariyal, Sunil Yadav and Sudhakar Kumar Chaubey *Ricci solitons on trans-Sasakian manifolds*, pp. 138-158. [PDF]
- 12. Absos Ali Shaikh, Akhlad Iqbal and Chandan Kumar Mondal Some results on phi-convex functions and geodesic phi-convex functions, pp. 159-170. [PDF]

# Some results on $\varphi$ -convex functions and geodesic $\varphi$ -convex functions

Absos Ali Shaikh, Akhlad Iqbal and Chandan Kumar Mondal

**Abstract.** As a generalization of geodesic function, in the present paper, we introduce the notion of geodesic  $\varphi$ -convex function and deduce some basic properties of  $\varphi$ -convex function and geodesic  $\varphi$ -convex function. We also introduce the concept of geodesic  $\varphi$ -convex set and  $\varphi$ -epigraph and investigate a characterization of geodesic  $\varphi$ -convex functions in terms of their  $\varphi$ -epigraphs.

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#### 1 Introduction

Convex sets and convex functions play an important role in the study of the theory of nonlinear programming and optimization. But in many situations only convexity is not enough to provide a satisfactory solution of a problem. Hence it is necessary to generalize the concept of convexity notion. Again, due to the curvature and torsion of a Riemannian manifold highly nonlinearity appears in the study of convexity on such a manifold. Geodesics are length minimizing curves, and the notion of geodesic convex function arises naturally on a complete Riemannian manifold and such a concept is investigated recently in ([13], [12], [4]).

In 2016 Eshaghi Gordji et. al. [3] defined the notion of  $\varphi$ -convex function and deduced Jensen and Hadamard type inequalities for such functions. In the present paper, we have obtained some other proerties of  $\varphi$ -convex functions. Again, generalizing the concept of  $\varphi$ -convex function, we have introduced the notion of *geodesic*  $\varphi$ -convex function on a complete Riemannian manifold and showed its existence by a proper example (see Example: 2.10). Convex sets on a Riemannian manifold have been generalized in different ways such as geodesic *E*-convex function [6], geodesic semi-*E*-convex function [7], geodesic semi *E*-*b*-vex functions [8] etc. We have also introduced a new class of sets, called, *geodesic*  $\varphi$ -convex sets on a complete Riemannian manifold.

The paper is organized as follows. Section 2 deals with the rudimentary facts of convex functions and convex sets. Section 3 is devoted to the study of some properties

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